

Inequality Issuization

The next page is about two inequality measures. These measures both compute an index for the inequality of the distribution of the quantity E over an area A. Here A also could represent the size of a group (amount of members), which is divided into several groups A_i where each subgroup (each fractile) has the resources (income or wealth etc.) E_i .

The *Hoover index* describes, how much resources would have been redistributed with minimum effort in order to have a perfectly equal distribution. Such a redistribution process would require absolute knowledge about the state of distribution at any time.

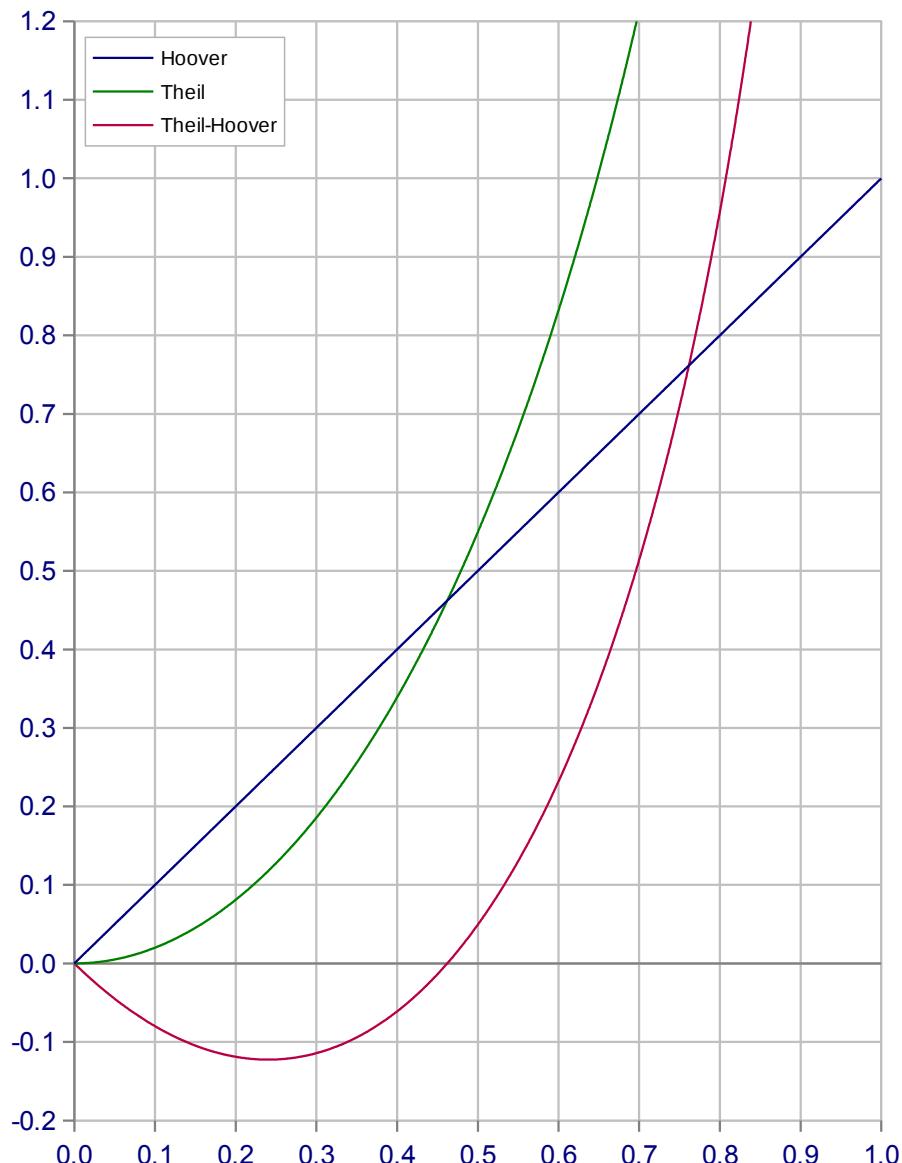
The *symmetrized Theil Index* is a *redundancy*. A redundancy is a negative entropy which describes the difference between maximum entropy (e.g. no differentiation between the groups due to perfectly equal distribution) in a system and the actual entropy in that system. This index could be applied to describe a state of a random equalization process like in an ideal gas. For the redistribution process knowledge about the state of distribution is required. It is perfectly stochastic.

The difference between both indices is *information*.

My proposal: The symmetrized Theil index is deducted from the Hoover index in order to find the largest information gap between both indices. At a Hoover index of around 24% the gap is at its maximum. I propose to test experimentally and empirically whether the inequality issuization in societies and test groups with unequal distributions of resources at around a Hoover index of 24% also is at its minimum. At around 46% both the gap is zero. Above around 46%, the symmetrized Theil index is larger than the Hoover index. I assume that above this threshold inequality causes significantly growing controversies due to higher awareness.

For this proposal, I split a group A into two a-fractiles A_1 and A_2 and distribute resources $E=E_1+E_2$ to them with $E_1=E \cdot A_2/A$ and $E_2=E \cdot A_1/A$ for various Hoover indices. For each E, A pair also a symmetrized Theil index is computed. See next page (written in 2009, with some little fixes done today).

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For $i = 1$ to n fractiles the E_i (all incomes per fractile) and the A_i (all earners per fractile) are used to compute the *Hoover Index* and the symmetrized *Theil Index*. The *Hoover Index* applies to minimum effort redistribution towards equilibrium. The *Theil Index* applies to stochastic redistribution towards equilibrium. I interpret the difference as "inequality issuization".

$$\begin{aligned}
 Z_{\text{Hoover}} &= \sum_{i=1 \dots n} (\text{abs}(E_i/E_{\text{total}} - A_i/A_{\text{total}})) / 2 \\
 Z_{\text{symmetricTheil}} &= \sum_{i=1 \dots n} (\ln(E_i/A_i) * (E_i/E_{\text{total}} - A_i/A_{\text{total}})) / 2 \\
 Z_{\text{inequalityIssuization}} &= Z_{\text{symmetricTheil}} - Z_{\text{Hoover}}
 \end{aligned}$$

The graphics refers to an income distribution of societies devided into two a-fractiles. Example: For a Hoover Index of 0.6 (or 60%), 80% earn 20% in the 1st a-fractile and 20% earn 80% in the 2nd a-fractile.

In case of a split into just two groups, the Hoover Index is similar to the Gini Index and $Z_{\text{symmetricTheil}} = 2 * Z_{\text{Hoover}} * \text{atanh}(Z_{\text{Hoover}})$ applies.