Inequality Issuization

The next page is about two inequality measures. These measures both compute an index for the inequality of the distribution of the quantity E over an area A. Here A also could represent the size of a group (amount of members), which is divided into several groups $A_i$ where each subgroup (each fractile) has the resources (income or wealth etc.) $E_i$.

The Hoover index describes, how much resources would have be redistributed with minimum effort in order to have a perfectly equal distribution. Such a redistribution process would require absolute knowledge about the state of distribution at any time.

The symmetrized Theil Index is an redundancy. A redundancy is a negative entropy which describes the difference between maximum entropy (e.g. no differentiation between the groups due to perfectly equal distribution) in a system and and the actual entropy in that system. This index could be applied to describe a state of a random equalization process like in an ideal gas. For the redistribution process would knowledge about the state of distribution is required. It is perfectly stochastic.

The difference between both indices is information.

My proposal: The symmetrized Theil index is deducted from the Hoover index in order to find the largest information gap between both indices. At a Hoover index of around 24% the gap is at its maximum. I propose to test experimentally and empirically whether the inequality issuization in societies an test groups with unequal distributions of resources at around a Hoover index of 24% also is at its minimum. At around 46% both the gap is zero. Above around 46%, the symmetrized Theil index is larger than the Hoover index. I assume that above this threshold inequality causes significantly growing controversies due to higher awareness.

For this proposal, I split a group A into two a-fractiles $A_1$ and $A_2$ and distribute resources $E=E_1+E_2$ to them with $E_1=E\cdot A_2/A$ and $E_2=E\cdot A_1/A$ for various Hoover indices. For each $E,A$ pair also a symmetrized Theil index is computed. See next page (written in 2009, with some little fixes done today).

Götz Kluge, 2015-01-31
For \(i = 1\) to \(n\) fractiles the \(E_i\) (all incomes per fractile) and the \(A_i\) (all earners per fractile) are used to compute the Hoover Index and the symmetrized Theil Index. The Hoover Index applies to minimum effort redistribution towards equilibrium. The Theil Index applies to stochastic redistribution towards equilibrium. I interpret the difference as "inequality issuization":

\[
Z_{\text{Hoover}} = \sum_{i=1}^{n} \left( \frac{\text{abs} (E_i/E_{\text{total}} - A_i/A_{\text{total}})}{2} \right)
\]

\[
Z_{\text{symmetricTheil}} = \sum_{i=1}^{n} \left( \frac{\ln(E_i/A_i) \times (E_i/E_{\text{total}} - A_i/A_{\text{total}})}{2} \right)
\]

\[
Z_{\text{inequalityIssuization}} = Z_{\text{symmetricTheil}} - Z_{\text{Hoover}}
\]

The graphics refers to an income distribution of societies divided into two a-fractiles. Example: For a Hoover Index of 0.6 (or 60%), 80% earn 20% in the 1st a-fractile and 20% earn 80% in the 2nd a-fractile.

In case of a split into just two groups, the Hoover Index is similar to the Gini Index and

\[
Z_{\text{symmetricTheil}} = 2 \times Z_{\text{Hoover}} \times \text{atanh}(Z_{\text{Hoover}})
\]

applies.